



Exam Problem Sheet

The exam consists of 6 problems. You can achieve 62 points in total.
 The number of points for each problem is marked in brackets (you get 6 points for free).
 You can find the translation of the problems into Dutch below.
 You may answer the questions in English or Dutch.

1. [3+3 Points] Let R be a ring, and $a \in R$. Define

$$S = \{x \in R : ax = xa\}.$$

- (a) Prove that S is a subring of R .
 (b) Prove: $S^* = R^* \cap S$.

2. [3+2+4 Points] Let V be a set, R a ring, and R^V the set of functions from V to R , i.e. $R^V = \{f : V \rightarrow R\}$.

- (a) Show that R^V is a ring when addition and multiplication are defined according to

$$\begin{aligned} (f+g)(v) &= f(v) + g(v), \\ (fg)(v) &= f(v) \cdot g(v) \end{aligned}$$

for $f, g : V \rightarrow R$ and $v \in R$.

- (b) Note that for $V = \{v_1, v_2, \dots, v_n\}$, with $n \in \mathbb{Z}_{>0}$, the ring R^V coincides with the product of rings $R \times R \times \dots \times R$ (n times). Show that for $n \geq 2$ and $R \neq \{0\}$, R^V always has zero divisors.
 (c) Let $V = [0, 1]$ and $R = \mathbb{R}$, and consider

$$C([0, 1]) = \{f : [0, 1] \rightarrow \mathbb{R} : f \text{ is continuous}\}.$$

Show that $C([0, 1])$ forms a subring of R^V , and $C([0, 1])$ has zero divisors.

3. [3+4+4 Points] Let

$$R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M(2, \mathbb{R}) : c = 0 \right\}$$

and

$$I = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \in M(2, \mathbb{R}) : b \in \mathbb{R} \right\}.$$

Prove the following statements:

- (a) R is a subring of $M(2, \mathbb{R})$;
 (b) I is an ideal of R , and $R/I \cong \mathbb{R} \times \mathbb{R}$;
 (c) R is not commutative, but R/I is commutative.

4. [3+4+7 Points]

- (a) What does the Chinese remainder theorem for rings say?
 (b) Let R be a unitary ring, and let I_1, I_2, I_3 be ideals of R . Show that if

$$I_1 + I_3 = R \text{ and } I_2 + I_3 = R$$

then

$$(I_1 \cdot I_2) + I_3 = R.$$

- (c) Let R be a commutative ring with 1, and let I_1, I_2, \dots, I_t ($t \in \mathbb{Z}_{>0}$) be ideals of R which are mutually prime, i.e. $I_i + I_j = R$ for $1 \leq i < j \leq t$. Prove that

$$R / \left(\prod_{i=1}^t I_i \right) \cong \prod_{i=1}^t (R / I_i).$$

(Hint: prove that $(I_1 \cdot I_2 \cdot \dots \cdot I_{t-1}) + I_t = R$ as in (b), and use induction by t . Use the Chinese remainder theorem for rings.)

5. [3 + 5 Points]

- (a) How are prime ideals and maximal ideals defined?
 (b) Let R be a domain. Prove: the ideal generated by X and Y in $R[X, Y]$ is equal to

$$\{f \in R[X, Y] : f(0, 0) = 0\}$$

(i.e. $(X, Y) = \{f \in R[X, Y] : f(0, 0) = 0\}$) and this is a prime ideal of $R[X, Y]$. Is this ideal also maximal?

6. [3 + 5 Points]

- (a) Give the definition of a unique factorization domain.
 (b) Determine all the irreducible polynomials $f \in \mathbb{F}_2[X]$ with $\deg(f) \leq 3$.

Dutch Translation

1. [3+3 Punten] Laat R een ring zijn, en $a \in R$. Definieer

$$S = \{x \in R : ax = xa\}.$$

- (a) Bewijs dat S een deelring van R is.
 (b) Bewijs: $S^* = R^* \cap S$.

2. [3+2+4 Punten] Zij V een verzameling, R een ring, en R^V de verzameling van afbeeldingen van V naar R , d.w.z. $R^V = \{f : V \rightarrow R\}$.

- (a) Bewijs dat R^V met de optelling and vermenigvuldiging

$$\begin{aligned} (f+g)(v) &= f(v) + g(v), \\ (fg)(v) &= f(v) \cdot g(v) \end{aligned}$$

voor $f, g : V \rightarrow R$ en $v \in R$, een ring vormt.

- (b) Geldt $V = \{v_1, v_2, \dots, v_n\}$, met $n \in \mathbb{Z}_{>0}$, dan zien we dat R^V dezelfde ring is als $R \times R \times \dots \times R$ (n keer). Bewijs dat voor $n \geq 2$ en $R \neq \{0\}$, R^V steeds nuldelers heeft.

(c) Zij $V = [0, 1]$ en $R = \mathbb{R}$, en beschouw

$$C([0, 1]) = \{f : [0, 1] \rightarrow \mathbb{R} : f \text{ is continu}\}.$$

Bewijs dat $C([0, 1])$ een deelring van R^V , en deze deelring heeft nog steeds nuldelers.

3. [3+4+4 Punten] Laat

$$R = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M(2, \mathbb{R}) : c = 0 \right\}$$

en

$$I = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \in M(2, \mathbb{R}) : b \in \mathbb{R} \right\}.$$

Bewijs de volgende uitspraken:

- (a) R is een deelring van $M(2, \mathbb{R})$;
- (b) I is een ideaal van R , en $R/I \cong \mathbb{R} \times \mathbb{R}$;
- (c) R is niet commutatief maar R/I wel.

4. [3+4+7 Punten]

- (a) Wat zegt de Chinese reststelling voor ringen?
- (b) Laat R een ring met 1 zijn, en I_1, I_2, I_3 idealen van R . Bewijs dat: Als

$$I_1 + I_3 = R \text{ en } I_2 + I_3 = R,$$

dan

$$(I_1 \cdot I_2) + I_3 = R.$$

- (c) Zij R een commutatieve ring met 1, en laten I_1, I_2, \dots, I_t ($t \in \mathbb{Z}_{>0}$) idealen van R zijn die paarsgewijs onderlig ondeelbaar zijn, d.w.z. $I_i + I_j = R$ voor $1 \leq i < j \leq t$. Bewijs:

$$R / \left(\prod_{i=1}^t I_i \right) \cong \prod_{i=1}^t (R / I_i).$$

(Aanwijzing: bewijs $(I_1 \cdot I_2 \cdot \dots \cdot I_{t-1}) + I_t = R$ als in (b), en pas inductie naar t toe. Maak gebruik van de Chinese reststelling voor ringen.)

5. [3 + 5 Punten]

- (a) Hoe zijn priem idealen en maximaal idealen gedefinieerd?
- (b) Laat R een domein zijn. Bewijs: het door X en Y voortgebrachte ideaal van $R[X, Y]$ is gelijk aan

$$\{f \in R[X, Y] : f(0, 0) = 0\}$$

(d.w.z. $(X, Y) = \{f \in R[X, Y] : f(0, 0) = 0\}$) en dit is een priemideaal van $R[X, Y]$. Is dit ideaal ook maximaal?

6. [3 + 5 Punten]

- (a) Geef de definitie van een ontbindingsdomein.
- (b) Bepaal alle irreducibele polynomen $f \in \mathbb{F}_2[X]$ met $\text{graad}(f) \leq 3$.

Tentamen Algebraïsche structuren bap'e 10

1^a $\forall a \in R$ (gegeven R ring)

$a \cdot 1 = a = 1 \cdot a$ dan $\forall a \in R$ dus $1 \in S$

$\forall a, b \in S$: $ax = xa$ $x \in R$

$bx = xb$ of $x \in S$ of $x \in R$

dan $ax - bx = xa - xb$ (distributieve wet)

$= (a-b)x = x(a-b)$

dus $a-b \in S$

$\forall a, b \in S$

$abx = a(bx) = a(xb) = (ax)b = xab$

$ab \in S$

3/3

Dus S is een deelring

$b \in R^*$ alle eenheden uit R dus alle $a, b \in R$ met

$ab = ba = 1$

\exists stel $c \in S^*$ dan $c \in S$ met de eigenschap $cd = dc = 1$ en $d \in S$, aangezien $c \in S$ geldt ook $c \in R$ ($\forall c \in R$) en zo ook $d \in R$, dus

$c, d \in R^*$ en dus $c \in R^*$ dus $c \in R^* \cap S$
 \exists stel $f \in R^* \cap S \rightarrow \exists e \in R^*$ zod $ef = fe = 1$ ($f \in R^*$), $f \in S$

Neem nu $e \in R$ hiervoor moet gelden

\otimes ~~$ef = fe = 1$~~
Dus ook $e \in S$

~~aangezien~~ $e, f \in S$ en $e, f \in R^*$ ~~igeldt~~
~~ook~~ $ef = fe = 1$ dus $e, f \in S^*$ ~~daar $\forall f \in R^* \cap S$~~

□

$E = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M(2, R) : c = 0 \}$
 $= \{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \in M(2, R) \}$

5/6

\ast $\begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix} \in R$ ($a=d=1, b=0$)

$\begin{pmatrix} e & b \\ g & h \end{pmatrix} \in M(2, R)$

$\begin{pmatrix} e & b \\ g & h \end{pmatrix} \begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix} = \begin{pmatrix} ea & ba \\ ga & ha \end{pmatrix} = \begin{pmatrix} e & b \\ g & h \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

dus $(0 \ 0) = 1_{M(2, \mathbb{R})} \in R \ \forall \begin{pmatrix} e & f \\ g & h \end{pmatrix} \in M(2, \mathbb{R})$

* ~~$\forall \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}, \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} \in R$~~

~~$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} = \begin{pmatrix} ae & af+bh \\ 0 & dh \end{pmatrix}$~~
 ~~$= \begin{pmatrix} a-e & b-f \\ 0 & d-h \end{pmatrix} \in R$~~

* ~~$\forall \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}, \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} \in R$~~

$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} = \begin{pmatrix} ae & af+bh \\ 0 & dh \end{pmatrix} \in R$

dus R is een deelring van $M(2, \mathbb{R})$

3/3

b * $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in I \ (b=0)$

* $\forall \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & c \\ 0 & 0 \end{pmatrix} \in I$

$\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & c \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & b-c \\ 0 & 0 \end{pmatrix} \in I$

* $\forall \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \in I$ en $\begin{pmatrix} g & e \\ 0 & + \end{pmatrix} \in R$

$\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \begin{pmatrix} g & e \\ 0 & + \end{pmatrix} = \begin{pmatrix} 0 & be \\ 0 & 0 \end{pmatrix} \in I$

$\begin{pmatrix} g & e \\ 0 & + \end{pmatrix} \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & gb \\ 0 & 0 \end{pmatrix} \in I$

dus I is een ideaal van R

✓

~~$\forall \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} \in I, \forall \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \in R, \forall \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} \in R, \forall \begin{pmatrix} g & e \\ 0 & + \end{pmatrix} \in R$~~

$R \rightarrow R/I \quad \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} - \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$

$\varphi: R/I \rightarrow R \times R$

* $\forall \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}, \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} \in R$

$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \begin{pmatrix} e & f \\ 0 & h \end{pmatrix} = \begin{pmatrix} ae & af+bh \\ 0 & dh \end{pmatrix} \neq \forall a, b, d, e, f, g, h$

$\begin{pmatrix} e & f \\ 0 & h \end{pmatrix} \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} = \begin{pmatrix} ea & eb+fd \\ 0 & hd \end{pmatrix}$

dus R niet commutatief (zie aan example!)

$$R/I \circ R \rightarrow R/I \quad a \rightarrow a + I$$

$$\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \mapsto \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \in \begin{pmatrix} R & 0 \\ 0 & R \end{pmatrix} + I$$

$$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}, \begin{pmatrix} e & 0 \\ 0 & h \end{pmatrix} \in R$$

be more precise!

$$\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} e & 0 \\ 0 & h \end{pmatrix} = \begin{pmatrix} ae & 0 \\ 0 & dh \end{pmatrix} = \begin{pmatrix} ea & 0 \\ 0 & hd \end{pmatrix} = \begin{pmatrix} e & 0 \\ 0 & h \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$$

$$a, b, e, d, h \in R$$

dus R/I is commutatief

$$b \quad \varphi\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\right) = (a, d)$$

3/4

$$\bullet \quad \varphi\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = (1, 1)$$

$$\bullet \quad \varphi\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} + \begin{pmatrix} c & 0 \\ 0 & b \end{pmatrix}\right) = \varphi\left(\begin{pmatrix} a+c & 0 \\ 0 & b+d \end{pmatrix}\right) = (a+c, b+d) \\ = (a, d) + (c, b) = \varphi\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\right) + \varphi\left(\begin{pmatrix} c & 0 \\ 0 & b \end{pmatrix}\right)$$

$$\bullet \quad \varphi\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} c & 0 \\ 0 & b \end{pmatrix}\right) = \varphi\left(\begin{pmatrix} ac & 0 \\ 0 & db \end{pmatrix}\right) = (ac, db) \\ = (a, d) \circ (c, b) = \varphi\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\right) \circ \varphi\left(\begin{pmatrix} c & 0 \\ 0 & b \end{pmatrix}\right)$$

Dus φ is een ringhomomorfisme

$$\text{Ker}(\varphi) = \text{Ker}(\varphi|_{M_2(R)}) = \text{Ker}(\varphi\left(\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}\right)) = \text{Ker}((a, d)) \\ = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in M_2(R), b \in R \right\} = I$$

Als φ surjectief, dan mbv de eerste isomorfiestelling geldt: $R/I \cong R \times R$

$$\forall (a, b) \in R \times R \quad \exists \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \in R \quad \text{met} \\ \varphi\left(\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\right) = (a, b) \quad \text{dus } \varphi \text{ surjectief}$$

□

4/4
10/11

φ^a Als $\cancel{R/I} \cdot \cancel{R/I} = \cancel{R/I}$ commutatieve ring met 1
 $\cancel{I, J}$ idealen van R en $I+J=R$

$$\cancel{I \cdot J = R}$$

$$\text{dus } R/(I \cdot J) \cong (R/I) \cdot (R/J)$$

$$\text{and } I \cdot J = I \cap J$$

11/11

5^a Priemideaal is een ideaal $p \in R$ met $p \in R^\times$
 als $p = ab$ dan $a = \pm 1$ of $b = \pm 1$ } u_{10}

6 Maximaal ideaal

Een ideaal $I \subset R$ is maximaal als $I \neq R$

en $\forall J$ ideaal J met

$$I \subset J \subset R \text{ geldt } J = I \text{ of } J = R$$

$$b \ R[x, y] = \sum_{j=0}^p \sum_{i=0}^m a_{ij} x^i y^j$$

$$I = \{ f \in R[x, y] : f(0,0) = 0 \}$$

$$g(0,0) = a_{00}$$

$$I = \{ f \in R[x, y] : a_{00} = 0 \}$$

$$* \ f = 0 \in I$$

$$* \ \forall f \in I \text{ en } g \in I:$$

$$f = \sum_{j=0}^p \sum_{i=0}^m a_{ij} x^i y^j ; a_{00} = 0$$

$$= \sum_{j=1}^p \sum_{i=0}^m a_{ij} x^i y^j + \sum_{i=1}^m a_{i0} x^i \in I$$

$$g = \sum_{e=1}^e \sum_{\beta=0}^e a_{\beta e} x^\beta y^e + \sum_{\beta=1}^e a_{\beta 0} x^\beta \in I$$

$$f - g = \sum_{j=1}^p \sum_{i=0}^m a_{ij} x^i y^j - \sum_{i=1}^m a_{i0} x^i - \sum_{e=1}^e \sum_{\beta=0}^e a_{\beta e} x^\beta y^e -$$

$$\sum_{\beta=1}^e a_{\beta 0} x^\beta$$

$$(f-g)(0,0) = 0 - 0 = 0 \in I$$

$$* \ \cancel{(fg)(0,0) = f(0,0) \cdot g(0,0) = 0 \cdot 0 = 0 \in I}$$

$$(fg)(0,0) = f(0,0) \cdot g(0,0) = 0 \cdot g(0,0) = 0 \in I$$

$$(gf)(0,0) = g(0,0) \cdot f(0,0) = g(0,0) \cdot 0 = 0 \in I$$

(2)

Tentamen Algebraïsche Structuren 6apere
5b dus I is een ideaal

^{Te bew}
dat $I = \{Xf + Yg \mid f, g \in R[x, y]\}$
(I0 wordt gebracht door x en y)

' \geq ' $(Xf + Yg)(0,0) = 0$ dus $Xf + Yg \in I$

' \leq ' $I = \{f \in R[x, y] : f(0,0) = 0\}$

$= \left\{ \sum_{j=0}^m \sum_{i=0}^m a_{ij} x^i y^j, a_{00} = 0 \right\}$

$= \left\{ \sum_{j=0}^m \sum_{i=0}^m a_{ij} x^i y^j + \sum_{i=0}^m a_{i0} x^i \right\}$

$= \left\{ Y \sum_{j=0}^m \sum_{i=0}^m a_{ij} x^i y^{j-1} + X \sum_{i=0}^m a_{i0} x^{i-1} \right\} = \{Yk + Xl \mid k, l \in R[x, y]\}$

dus $I = \{Xf + Yg, f, g \in R[x, y]\} = (x, y)$

~~R is een domain, I priem ideaal, R/I is een lichaam~~

~~I is een maximaal ideaal~~

$x = 0$ en $y = 0$ zijn priem in $R[x, y]$, dus (x, y)

is een priemideaal. $R/(x, y) \cong R$ is een lichaam is (x, y) maximaal

$R[x, y]/(x, y) \cong R$ die is een ring
er moet gelden $\forall a, b \in R \quad ab = ba$

~~$a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$~~ $b = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ ($R = M(2, R)$)
 $ab = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $ba = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

dus R is geen lichaam (niet commutatief) dus (x, y) is R niet maximaal
(voor $R = R$ is (x, y) maximaal want R is een lichaam)

R is a domain
in this exercise!

2.5/5

6^a R een domain, $\forall a \in R, a \neq 0$ kan a gescheven worden als een product van een eenheid en een eindig aantal irreducibele factoren, die product is op volgorde na een *and unit factors!*

b $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z} = \{0, 1\}$
 $f \in \mathbb{F}_2$ def $g(f) \leq 3$

$\frac{2}{3}$

$$f = a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

als de nulpunten van f niet in \mathbb{F}_2 liggen, dan is f irreducibel dus $f(0) \neq 0$ en $f(1) \neq 0$

$$f(0) = a_0$$

$$f(1) = a_3 + a_2 + a_1 + a_0$$

$$\text{Dus } a_0 = 1 (\neq 0)$$

$$a_3 + a_2 + a_1 + a_0 = 1 (\neq 0) \text{ (dus of } a_0 = 1 \text{ of } a_0 \text{ en } a_1 \text{ en } a_2 \text{ en } a_3 \text{ zijn } 1)$$

Dus de irreducibele polynomen zijn:

~~$$f = \{ a_0 + a_1 x + a_2 x^2, a_0 + a_1 x + a_2 x^2 \}$$~~

$$f = \{ 1, 1 + x + x^2, 1 + x + x^3, 1 + x^2 + x^3 \}$$

$\frac{25}{15}$
 $\frac{45}{8}$

2^a R^v $\forall f, g \in R^v$
 $(f+g)(v) = (a_0+0)(v) = a_1(v) + 0 = (0+a_1)(v)$

$$\bullet (f+g)(v) = f(v) + g(v) = g(v) + f(v) = (g+f)(v)$$

$$\bullet \forall f \in R^v \exists -f \in R^v \text{ zod}$$

$$(f+(-f))(v) = f(v) - f(v) = 0 = (-f+f)(v)$$

$$\bullet (f+g)+h(v) = (f+g)(v) + h(v) = f(v) + g(v) + h(v)$$

$$= f(v) + (g+h)(v) = (f+(g+h))(v)$$

$$\times \forall f, g, h \in R^v$$

$$(f(gh))(v) = f(v) \cdot (gh)(v) = f(v) g(v) h(v) = (fg)(v) h(v) = ((fg)h)(v)$$

$$\times \exists 1 \in R^v \text{ met } (f \circ 1)(v) = f(v) = (f \cdot g)(v) = g(v) f(v) = 1 f(v)$$

$$\times (f(g+h))(v) = f(v) (g+h)(v) = f(v) g(v) + f(v) h(v) = (fg)(v) + (fh)(v)$$

$$((g+h)f)(v) = (g+h)(v) f(v) = g(v) f(v) + h(v) f(v)$$

$$= (gf)(v) + (hf)(v) \quad \forall f, g, h \in R^v$$

dus R^v is een ring

b^a n=2

$$(1,0) \times (0,1) = (0 \cdot 1, 0 \cdot 0) = (0,0)$$

° (1,0) en (0,1) zijn niet 0 dus zijn nuldeleers

voor n=i ≥ 2

$$(*) = \underbrace{(1,0,\dots,0)}_{\text{elementen}} \times (0,1,0,\dots,0) \times \dots \times (0,\dots,0,1) \leftarrow \text{alle } n \text{ ai on gelijk aan } 0$$

$$= (1 \cdot 0 \cdot 0 \cdot \dots \cdot 0, 0 \cdot 1 \cdot 0 \cdot \dots \cdot 0, \dots, 0 \cdot \dots \cdot 0 \cdot 1)$$

= (0,0,...,0) nulelement, dus R^v heeft

~~n-1~~ nuldeleers

$$\text{and } (0,\dots,0,1,0,\dots,0)$$

van ter overal 0 behalve op de j^e plek

$$(*) = \prod_{j=1}^i (0, \dots, 0, \underbrace{1}_{\text{j^e plek}}, \dots, 0) = (0,0,\dots,0)$$

a

Dus voor i ≥ 2 heeft R^v altijd nuldeleers

2/2

~~via~~

$$c \times f(k) = 0 \quad \forall k \in C(0,1]$$

dus 0 ∈ C(0,1]

$$* \forall f, g \in C(0,1]$$

f-g ∈ continu (f, g continu)

$$f-g \in C(0,1] \rightarrow \mathbb{R} \quad (f, g \in C(0,1] \rightarrow \mathbb{R})$$

dus f-g ∈ C(0,1] ∀ f, g ∈ C(0,1]

$$* \forall f, g \in C(0,1]$$

fg continu (f, g continu)

$$fg \in C(0,1] \rightarrow \mathbb{R} \quad (f, g \in C(0,1] \rightarrow \mathbb{R})$$

dus fg ∈ C(0,1] ∀ f, g ∈ C(0,1]

en dus is C(0,1] een deelring van R^v

V=C(0,1] dus n ≥ 2 (nops dus R^v ∈ C(0,1])

C(1,0] heeft nuldeleers (zie b)

-2/4

C(0,1] does not have finitely many

$$\begin{aligned}
 4^b \quad I_1 &= (a_1, \dots, a_n) \\
 I_2 &= (b_1, \dots, b_m) \\
 I_3 &= (c_1, \dots, c_\ell) \\
 I_1 + I_3 &= (a_1, \dots, a_n) + (c_1, \dots, c_\ell) \\
 &= \sum_{i=1}^n \sum_{j=1}^{\ell} (a_i + c_j)
 \end{aligned}$$

$$\begin{aligned}
 I_2 + I_3 &= (b_1, \dots, b_m) + (c_1, \dots, c_\ell) \\
 &= \sum_{i=1}^m \sum_{j=1}^{\ell} (b_i + c_j)
 \end{aligned}$$

$$I_1 + I_3 = I_2 + I_3 = R$$

$$\begin{aligned}
 I_1 \cdot I_2 &= (a_1, \dots, a_n) \cdot (b_1, \dots, b_m) \\
 &= \sum_{i=1}^n a_i \sum_{j=1}^m (a_i b_j)
 \end{aligned}$$

~~3~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ ~~137~~ 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~~388~~ ~~389~~ ~~390~~ ~~391~~ ~~392~~ ~~393~~ ~~394~~ ~~395~~ ~~396~~ ~~397~~ ~~398~~ ~~399~~ ~~400~~ ~~401~~ ~~402~~ ~~403~~ ~~404~~ ~~405~~ ~~406~~ ~~407~~ ~~408~~ ~~409~~ ~~410~~ ~~411~~ ~~412~~ ~~413~~ ~~414~~ ~~415~~ ~~416~~ ~~417~~ ~~418~~ ~~419~~ ~~420~~ ~~421~~ ~~422~~ ~~423~~ ~~424~~ ~~425~~ ~~426~~ ~~427~~ ~~428~~ ~~429~~ ~~430~~ ~~431~~ ~~432~~ ~~433~~ ~~434~~ ~~435~~ ~~436~~ ~~437~~ ~~438~~ ~~439~~ ~~440~~ ~~441~~ ~~442~~ ~~443~~ ~~444~~ ~~445~~ ~~446~~ ~~447~~ ~~448~~ ~~449~~ ~~450~~ ~~451~~ ~~452~~ ~~453~~ ~~454~~ ~~455~~ ~~456~~ ~~457~~ ~~458~~ ~~459~~ ~~460~~ ~~461~~ ~~462~~ ~~463~~ ~~464~~ ~~465~~ ~~466~~ ~~467~~ ~~468~~ ~~469~~ ~~470~~ ~~471~~ ~~472~~ ~~473~~ ~~474~~ ~~475~~ ~~476~~ ~~477~~ ~~478~~ ~~479~~ ~~480~~ ~~481~~ ~~482~~ ~~483~~ ~~484~~ ~~485~~ ~~486~~ ~~487~~ ~~488~~ ~~489~~ ~~490~~ ~~491~~ ~~492~~ ~~493~~ ~~494~~ ~~495~~ ~~496~~ ~~497~~ ~~498~~ ~~499~~ ~~500~~ ~~501~~ ~~502~~ ~~503~~ ~~504~~ ~~505~~ ~~506~~ ~~507~~ ~~508~~ ~~509~~ ~~510~~ ~~511~~ ~~512~~ ~~513~~ ~~514~~ ~~515~~ ~~516~~ ~~517~~ ~~518~~ ~~519~~ ~~520~~ ~~521~~ ~~522~~ ~~523~~ ~~524~~ ~~525~~ ~~526~~ ~~527~~ ~~528~~ ~~529~~ ~~530~~ ~~531~~ ~~532~~ ~~533~~ ~~534~~ ~~535~~ ~~536~~ ~~537~~ ~~538~~ ~~539~~ ~~540~~ ~~541~~ ~~542~~ ~~543~~ ~~544~~ ~~545~~ ~~546~~ ~~547~~ ~~548~~ ~~549~~ ~~550~~ ~~551~~ ~~552~~ ~~553~~ ~~554~~ ~~555~~ ~~556~~ ~~557~~ ~~558~~ ~~559~~ ~~560~~ ~~561~~ ~~562~~ ~~563~~ ~~564~~ ~~565~~ ~~566~~ ~~567~~ ~~568~~ ~~569~~ ~~570~~ ~~571~~ ~~572~~ ~~573~~ ~~574~~ ~~575~~ ~~576~~ ~~577~~ ~~578~~ ~~579~~ ~~580~~ ~~581~~ ~~582~~ ~~583~~ ~~584~~ ~~585~~ ~~586~~ ~~587~~ ~~588~~ ~~589~~ ~~590~~ ~~591~~ ~~592~~ ~~593~~ ~~594~~ ~~595~~ ~~596~~ ~~597~~ ~~598~~ ~~599~~ ~~600~~ ~~601~~ ~~602~~ ~~603~~ ~~604~~ ~~605~~ ~~606~~ ~~607~~ ~~608~~ ~~609~~ ~~610~~ ~~611~~ ~~612~~ ~~613~~ ~~614~~ ~~615~~ ~~616~~ ~~617~~ ~~618~~ ~~619~~ ~~620~~ ~~621~~ ~~622~~ ~~623~~ ~~624~~ ~~625~~ ~~626~~ ~~627~~ ~~628~~ ~~629~~ ~~630~~ ~~631~~ ~~632~~ ~~633~~ ~~634~~ ~~635~~ ~~636~~ ~~637~~ ~~638~~ ~~639~~ ~~640~~ ~~641~~ ~~642~~ ~~643~~ ~~644~~ ~~645~~ ~~646~~ ~~647~~ ~~648~~ ~~649~~ ~~650~~ ~~651~~ ~~652~~ ~~653~~ ~~654~~ ~~655~~ ~~656~~ ~~657~~ ~~658~~ ~~659~~ ~~660~~ ~~661~~ ~~662~~ ~~663~~ ~~664~~ ~~665~~ ~~666~~ ~~667~~ ~~668~~ ~~669~~ ~~670~~ ~~671~~ ~~672~~ ~~673~~ ~~674~~ ~~675~~ ~~676~~ ~~677~~ ~~678~~ ~~679~~ ~~680~~ ~~681~~ ~~682~~ ~~683~~ ~~684~~ ~~685~~ ~~686~~ ~~687~~ ~~688~~ ~~689~~ ~~690~~ ~~691~~ ~~692~~ ~~693~~ ~~694~~ ~~695~~ ~~696~~ ~~697~~ ~~698~~ ~~699~~ ~~700~~ ~~701~~ ~~702~~ ~~703~~ ~~704~~ ~~705~~ ~~706~~ ~~707~~ ~~708~~ ~~709~~ ~~710~~ ~~711~~ ~~712~~ ~~713~~ ~~714~~ ~~715~~ ~~716~~ ~~717~~ ~~718~~ ~~719~~ ~~720~~ ~~721~~ ~~722~~ ~~723~~ ~~724~~ ~~725~~ ~~726~~ ~~727~~ ~~728~~ ~~729~~ ~~730~~ ~~731~~ ~~732~~ ~~733~~ ~~734~~ ~~735~~ ~~736~~ ~~737~~ ~~738~~ ~~739~~ ~~740~~ ~~741~~ ~~742~~ ~~743~~ ~~744~~ ~~745~~ ~~746~~ ~~747~~ ~~748~~ ~~749~~ ~~750~~ ~~751~~ ~~752~~ ~~753~~ ~~754~~ ~~755~~ ~~756~~ ~~757~~ ~~758~~ ~~759~~ ~~760~~ ~~761~~ ~~762~~ ~~763~~ ~~764~~ ~~765~~ ~~766~~ ~~767~~ ~~768~~ ~~769~~ ~~770~~ ~~771~~ ~~772~~ ~~773~~ ~~774~~ ~~775~~ ~~776~~ ~~777~~ ~~778~~ ~~779~~ ~~780~~ ~~781~~ ~~782~~ ~~783~~ ~~784~~ ~~785~~ ~~786~~ ~~787~~ ~~788~~ ~~789~~ ~~790~~ ~~791~~ ~~792~~ ~~793~~ ~~794~~ ~~795~~ ~~796~~ ~~797~~ ~~798~~ ~~799~~ ~~800~~ ~~801~~ ~~802~~ ~~803~~ ~~804~~ ~~805~~ ~~806~~ ~~807~~ ~~808~~ ~~809~~ ~~810~~ ~~811~~ ~~812~~ ~~813~~ ~~814~~ ~~815~~ ~~816~~ ~~817~~ ~~818~~ ~~819~~ ~~820~~ ~~821~~ ~~822~~ ~~823~~ ~~824~~ ~~825~~ ~~826~~ ~~827~~ ~~828~~ ~~829~~ ~~830~~ ~~831~~ ~~832~~ ~~833~~ ~~834~~ ~~835~~ ~~836~~ ~~837~~ ~~838~~ ~~839~~ ~~840~~ ~~841~~ ~~842~~ ~~843~~ ~~844~~ ~~845~~ ~~846~~ ~~847~~ ~~848~~ ~~849~~ ~~850~~ ~~851~~ ~~852~~ ~~853~~ ~~854~~ ~~855~~ ~~856~~ ~~857~~ ~~858~~ ~~859~~ ~~860~~ ~~861~~ ~~862~~ ~~863~~ ~~864~~ ~~865~~ ~~866~~ ~~867~~ ~~868~~ ~~869~~ ~~870~~ ~~871~~ ~~872~~ ~~873~~ ~~874~~ ~~875~~ ~~876~~ ~~877~~ ~~878~~ ~~879~~ ~~880~~ ~~881~~ ~~882~~ ~~883~~ ~~884~~ ~~885~~ ~~886~~ ~~887~~ 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